

Maths-Whizz Workshop

We have chosen a problem that has relatively easy entry points so that all students can begin, but also has scope for exploration and challenge for students at all levels. The problem will allow students to think, reason and make decisions – in other words, to work like mathematicians at a level and pace appropriate to them.

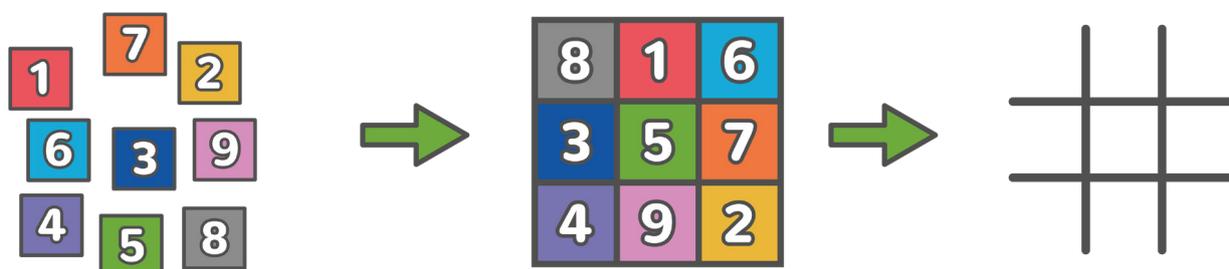
Sum to 15

Two players have number cards 1 – 9 spaced out between them. The players take it in turns to pick a number card. The idea of the game is to be the first to make the sum of 15 with three cards. The complexity comes in finding the winning strategy... - the choice of numbers made at each turn - whether it matters who goes first - what are all the winning combinations?

After a few games you should ask students - what does the game remind you of? Some students are likely to pick up the similarities with tic-tac-toe:

- a winning set consists of three elements
- some moves block potential sets from your opponent
- sometimes a player wins by setting up two winning sets, only one of which could be blocked
- some games end in a tie

To make the link between our game and tic-tac-toe, we can use magic squares!



Magic Squares

Place the nine numbers into a 3 x 3 grid so the rows, columns and diagonals have the same total.

Can you work out the 'magic number' before placing the numbers into the grid?

The magic number can be found by adding all the numbers in the square and dividing by 3 (as it's a 3 x 3 grid) and the total is shared equally across 3 rows, columns and diagonal.

There is also magic to be found in the symmetry of how all the lines add up to the same amount.

There are six different symmetries of a 3 x 3 magic square with the same set of numbers.

Take the learning even further - In 3 x 3 grid is the centre number always a 1/3 of the magic number?

Have a go at exploring the Gnomon Magic Square (4 x 4 grid). This magic square has the additional property that the sums in any of the four quadrants, as well as the sum of the middle four numbers, are all 34.

Take a look at its special properties.

Is there a winning strategy?

Winning in Sum to 15 is the same as finding a straight line in a 3x3 grid, which makes our game equivalent to tic-tac-toe. They are two representations of the same game. Sum to 15 is Tic-tac-toe in disguise. Who'd have thought it?!

So, what's the answer to our original question? Is there a strategy for winning in Sum to 15? Well, you probably know that tic-tac-toe has no guaranteed strategy – if both players know the handful of combinations in the game and each play a 'perfect' strategy, neither will win. So, there is no guaranteed strategy for Sum to 15 either!

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Some variants of magic squares

- 'un-magic' squares using digits 1 – 9 the sum of the rows, columns and diagonals must be different.
- 'prime' squares – using digits 1 – 9 all the rows and columns must add up to prime numbers. Is it possible to have the diagonal as well?

Try some of the tic-tac-toe variants below

Dotty Six (Nrich)

<https://nrich.maths.org/7337>

Traffic lights (Nrich)

<https://nrich.maths.org/1181>

Tic-tac-toe

<https://nrich.maths.org/538>

The Ultimate Tic-tac-toe

<https://mathwithbaddrawings.com/2013/06/16/ultimate-Tic-tac-toe/>

Technology can play an important part throughout this problem from teacher demonstrating, to gathering student feedback and exploring the suggested variants on Magic squares or Tic Tac Toe.

You can hopefully see why we choose this rich problem.

We started with a simple game of addition and ending up in the realm of tic-tac-toe, a game without numbers!

We found a surprising connection between two very different looking ideas by navigating the mystery of magic squares. Maths is all about finding these hidden connections.

There are downloadable resources to accompany this problem. Take a look at our other Extended Problems on Maths-Whizz Teachers' Resource.